

NAG Fortran Library Routine Document

S11ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S11ABF returns the value of the inverse hyperbolic sine, $\operatorname{arcsinh} x$, via the routine name.

2 Specification

```

real FUNCTION S11ABF(X, IFAIL)
  INTEGER          IFAIL
  real            X

```

3 Description

The routine calculates an approximate value for the inverse hyperbolic sine of its argument, $\operatorname{arcsinh} x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\operatorname{arcsinh} x = x \times y(t) = x \sum_{r=0}^{\prime} c_r T_r(t), \quad \text{where } t = 2x^2 - 1.$$

For $|x| > 1$ it uses the fact that

$$\operatorname{arcsinh} x = \operatorname{sign} x \times \ln\left(|x| + \sqrt{x^2 + 1}\right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2 + 1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arcsinh} x = \operatorname{sign} x \times (\ln 2 + \ln |x|).$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – *real* *Input*
On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{1+x^2} \operatorname{arcsinh} x} \delta \right|.$$

That is, the relative error in the argument, x , is amplified by a factor at least $\frac{x}{\sqrt{1+x^2} \operatorname{arcsinh} x}$, in the result.

The equality should hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

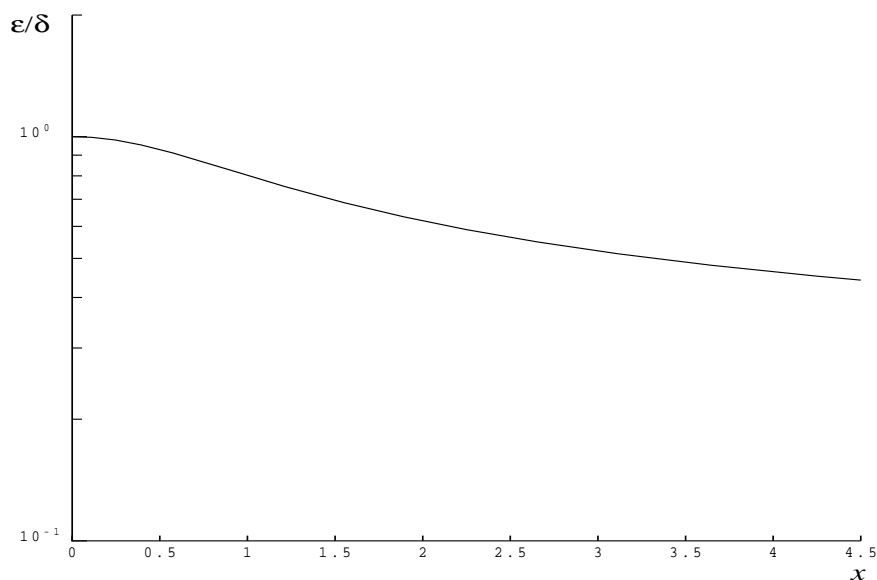


Figure 1

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E , in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by *machine precision*.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      S11ABF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
real           X, Y
INTEGER         IFAIL
*      .. External Functions ..
real          S11ABF
EXTERNAL        S11ABF
*      .. Executable Statements ..
WRITE (NOUT,*) 'S11ABF Example Program Results'
*      Skip heading in data file
READ (NIN,*)
WRITE (NOUT,*)
WRITE (NOUT,*) '      X          Y          IFAIL'
WRITE (NOUT,*)
20 READ (NIN,*,END=40) X
   IFAIL = 1
*
   Y = S11ABF(X,IFAIL)
*
   WRITE (NOUT,99999) X, Y, IFAIL
   GO TO 20
40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
END
```

9.2 Program Data

```
S11ABF Example Program Data
      -2.0
      -0.5
      1.0
      6.0
```

9.3 Program Results

```
S11ABF Example Program Results
```

X	Y	IFAIL
-2.000E+00	-1.444E+00	0
-5.000E-01	-4.812E-01	0
1.000E+00	8.814E-01	0
6.000E+00	2.492E+00	0
